

DESIGN OF SLIDING CONTROLLER USING LYAPUNOV STABILITY THEORY FOR CONTROL OF DOUBLE PENDULUM GANTRY CRANE

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ABSTRACT:

The difficulty in controlling the operation of the gantry crane (GC) is to face the complex phenomenon of the double pendulum. The oscillation of the hook (H) and the load (L) has reduced the working efficiency, reduced the ability to accurately position the trolley (T) and even caused mechanical damage and unsafe accidents. Therefore, the paper presents a solution to design a sliding mode controller (SMC) to control the gantry crane to reduce the oscillation of the hook and the load, and at the same time increase the ability to position the forklift. The coefficient k_s is optimized according to the fuzzy controller (FC). The stability of the system is proven by the Lyapunov stability theory, which has achieved the accuracy and durability of the entire control system. The sliding mode controller has been tested through MATLAB/Simulink simulation. The simulation results show that when using the sliding mode controller, the system has good control quality.

Keywords: Gantry crane; Sliding mode control; Position control; Oscillation control; Fuzzy control.

1. INTRODUCTION

Double pendulum gantry crane is a smart and popular mechanical design used worldwide in shipping yards, seaports, construction sites, factories and other industrial areas. This is a particularly important device for lifting, lowering, transporting and assembling goods and materials at high altitudes, in difficult conditions and transporting large machinery parts and heavy materials in limited spaces. To operate GC optimally, effectively and safely, it is necessary to build an optimal controller that can control three parameters well: the H swing angle (first swing angle), the L swing angle (second swing angle) and the T position (vehicle running along the beam). Therefore, there have been many studies on dynamic models and application of control methods to improve the performance of GC.

Structurally, GC is moved by T running along the beam, H is suspended on T and L is suspended on H via suspension cables [1], [2], [3], these structures are as shown in Fig. 1. GC has the functions of lifting, lowering and moving, however, when operating, the natural

swing angle of the first swing angle and the second swing angle makes the lifting, lowering and moving functions operate inefficiently, which is a complex double pendulum-like motion [4],[5].

The continuous oscillation of H and L is due to the increasing and decreasing speed of T and the impact of noise such as friction, wind, collision... Therefore, some major studies are used to control GC operations automatically such as input shape [6], adaptive control [7], FC-PID [8], [9] combines the advantages of PID when the system is approaching the setpoint and the advantage of FC that it works very well at large deviations, its nonlinearity can produce a very fast response. The authors used PSO [10], DE [11], GA [12], [13] algorithms to find the optimal parameters of PID for GC, achieving small shaking angle, however, the stability when there is external noise is not high. In addition, FC techniques have shown successful results in practical applications, including GC [14], double FC [15] which have the advantage of achieving small swing angles but have large POT overshoot

and settling time. SMC [16], SMC-FC [17], [18] have the advantage of achieving stability and sustainability even when there is disturbance affecting the GC, however, the new control algorithms stop at controlling the GC with a simple pendulum type. Therefore, in this paper, a dynamic model of a double pendulum GC is proposed, from which the SMC is designed with the coefficient k_s optimized through FC to control the position of T, control the swing angle of H and the swing angle of L. The stability of GC is proved by Lyapunov stability theory. Using MATLAB/Simulink simulation, the designed SMC is verified to have good working results.



Figure 1. Image of the gantry crane

The remaining sections of the paper are structured as follows: Section 2 presents the dynamic model of the double pendulum gantry crane system. The design of the sliding controller is presented in Section 3. Section 4 describes the simulation results. Section 5 concludes.

2. DYNAMIC MODEL OF THE DOUBLE PENDULUM GANTRY CRANE SYSTEM

The double pendulum gantry crane is shown in Figure 2, the technical parameters and values are taken according to the actual values as shown in table 1. GC can be modeled as a forklift with mass of T, mass of H, mass of L respectively M, m_1, m_2 , Hook cable length, load cable length respectively l_1, l_2 . The oscillation angle of the hook and the oscillation angle of the load are respectively y_1, y_2 . The angular velocity of hook H and the angular velocity of load L are \dot{y}_1, \dot{y}_2 respectively. The GC moves with a pushing force $F(N)$, q_d is the

external disturbances acting on the GC. Assume the cable is rigid and massless.

The equations of motion can be obtained by the Lagrangian equation [17]:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad (1)$$

Where: Q_j is the external force, q_j is the generalized coordinate system, j is the number of degrees of freedom of GC, $L = W_d - W_t$, W_t is the potential energy of GC and W_d is the kinetic energy of GC:

$$W_d = \sum_{i=1}^n \frac{1}{2} m_i \dot{x}_i^2 \quad (2)$$

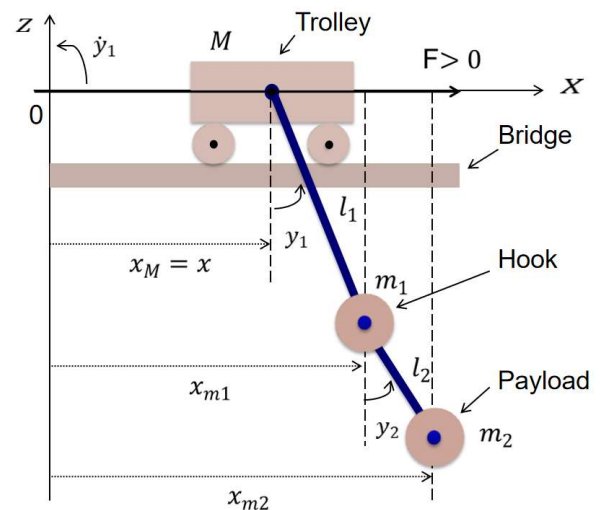


Figure 2. Schematic diagram of the GC system

Table 1. Symbols and values of GC parameters

| Symbol | Describe | Value | Unit |
|--------|------------------------------|-------|------------------|
| M | Weight of trolley | 24000 | Kg |
| m_1 | Weight of hook | 7000 | Kg |
| m_2 | Weight of electrolytic plate | 10000 | Kg |
| l_1 | Length of cable hook | 4 | m |
| l_2 | Length of electrolytic plate | 1.2 | m |
| g | Gravitational constant | 9,81 | m/s ² |
| μ | Coefficient of friction | 0,2 | N/m/s |

Figure 2 shows the position and velocity components of the forklift as follows:

$$x_M = x \quad (3)$$

$$\dot{x}_M = \dot{x} \quad (4)$$

The position and velocity components of the hook are:

$$x_{m1} = x + l_1 s(y_1) \quad (5)$$

$$\dot{x}_{m1} = \dot{x} + l_1 \dot{y}_1 c(y_1) \quad (6)$$

Where: $s(y_1) = \sin y_1$; $c(y_1) = \cos y_1$

The position and velocity components of the load are:

$$x_{m2} = x + l_1 s(y_1) + l_2 s(y_2) \quad (7)$$

$$\dot{x}_{m2} = \dot{x} + l_1 \dot{y}_1 c(y_1) + l_2 \dot{y}_2 c(y_2) \quad (8)$$

Where: $s(y_2) = \sin y_2$; $c(y_2) = \cos y_2$

The kinetic energy of the forklift, hook and load is:

$$W_{dM} = \frac{1}{2} M \dot{x}^2 \quad (9)$$

$$W_{dm1} = \frac{1}{2} m_1 (\dot{x}^2 + l_1^2 \dot{y}_1^2 + 2\dot{x}l_1\dot{y}_1 c(y_1)) \quad (10)$$

$$W_{dm2} = \frac{1}{2} m_2 (\dot{x}^2 + l_1^2 \dot{y}_1^2 + l_2^2 \dot{y}_2^2 + 2\dot{x}l_1\dot{y}_1 c(y_1) + 2\dot{x}l_2\dot{y}_2 c(y_2) + 2l_1l_2\dot{y}_1\dot{y}_2 c(y_1 - y_2)) \quad (11)$$

From (9), (10), (11) it follows that the kinetic energy of GC is:

$$\begin{aligned} W_d &= W_{dM} + W_{dm1} + W_{dm2} = \frac{1}{2} M \dot{x}^2 \\ &+ \frac{1}{2} m_1 (\dot{x}^2 + l_1^2 \dot{y}_1^2 + 2\dot{x}l_1\dot{y}_1 c(y_1)) \\ &+ \frac{1}{2} m_2 (\dot{x}^2 + l_1^2 \dot{y}_1^2 + l_2^2 \dot{y}_2^2 + 2\dot{x}l_1\dot{y}_1 c(y_1) \\ &+ 2\dot{x}l_2\dot{y}_2 c(y_2) + 2l_1l_2\dot{y}_1\dot{y}_2 c(y_1 - y_2)) \end{aligned} \quad (12)$$

Where: $c(y_1 - y_2) = \cos(y_1 - y_2)$

The potential energy of GC is:

$$\begin{aligned} W_t &= (m_1 + m_2)gl_1(1 - c(y_1)) \\ &+ m_2gl_2(1 - c(y_2)) \end{aligned} \quad (13)$$

By substituting (12) and (13) into (1), the following nonlinear motion equation of GC is derived [2]:

$$M_t \ddot{x} + M_{12}l_1(\ddot{y}_1 c(y_1) - \dot{y}_1^2 s(y_1)) + m_2l_2(\ddot{y}_2 c(y_2) - \dot{y}_2^2 s(y_2)) = F + q_d - \mu \dot{x} \quad (14)$$

$$M_{12}l_1 c(y_1) \ddot{x} + M_{12}l_1^2 \ddot{y}_1 + m_2l_1l_2 \ddot{y}_2 c(y_1 - y_2) + m_2l_1l_2 \dot{y}_2^2 s(y_1 - y_2) + M_{12}gl_1 s(y_1) = 0 \quad (15)$$

$$m_2l_2 c(y_2) \ddot{x} + m_2l_1l_2 \ddot{y}_1 c(y_1 - y_2) + m_2l_2^2 \ddot{y}_2 - m_2l_1l_2 \dot{y}_1^2 s(y_1 - y_2) + m_2gl_2 s(y_2) = 0 \quad (16)$$

Where: $M_t = M + m_1 + m_2$; $M_{12} = m_1 + m_2$; $s(y_1 - y_2) = \sin(y_1 - y_2)$

From (14), (15), (16) to lower the derivative of the nonlinear equation of GC, let $F = u$; $x_1 = x$; $x_2 = \dot{x}$; $x_3 = y_1$; $x_4 = \dot{y}_1$; $x_5 = y_2$; $x_6 =$

\dot{y}_2 The system of equations of motion of GC is given as follows [17]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(X(t)) + b_1(X(t))u + q_{d1}(X(t)) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(X(t)) + b_2(X(t))u + q_{d2}(X(t)) \\ \dot{x}_5 &= x_6 \\ \dot{x}_6 &= f_3(X(t)) + b_3(X(t))u + q_{d3}(X(t)) \end{aligned} \quad (17)$$

Where: $X(t) = [x_1, x_2, x_3, x_4, x_5, x_6]^T$ is the vector of position and speed of the crane, angle, angular velocity of the hook and load., $f_1(X(t)), f_2(X(t)), f_3(X(t)), b_1(X(t)), b_2(X(t)),$

$b_3(X(t))$ are non-linear functions, $q_{d1}(X(t)), q_{d2}(X(t)), q_{d3}(X(t))$ are the outer bound disturbances of wind and collision, u is the GC control input signal.

The dynamic model of GC that the authors proposed to design has some differences from the mathematical model in the published study [17] specifically as follows: The mathematical model of the system in the study [17] is a simple pendulum type GC control model, in the dynamic model of GC there are no external components of wind and collision. $q_{d1}(X(t)), q_{d2}(X(t)), q_{d3}(X(t))$ and functions $f_1(X(t)), f_2(X(t)), b_1(X(t)), b_2(X(t))$ is a linear function.

Based on (17), the GC dynamic model diagram is constructed as shown in Figure 3.

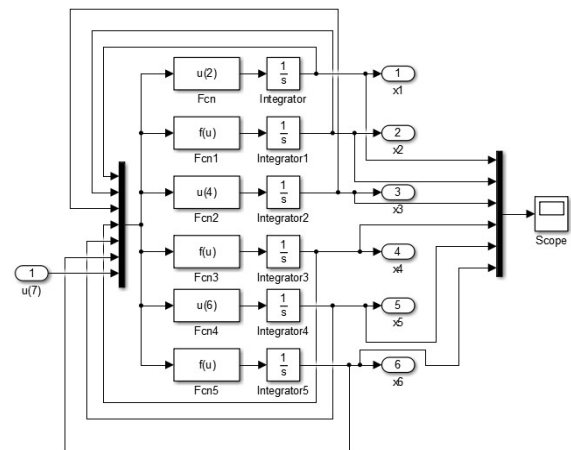


Figure 3. Matlab structure diagram of GC

Using MATLAB/Simulink software to simulate the designed GC dynamic model in the absence of control devices and without external disturbances affecting the system to

verify the accuracy of the GC dynamic model, at the same time, it lays a solid foundation for research, calculations, and experimental design in this model. The dynamic model is simulated with the system parameters used in Table 1 and $u = 1000000N$. The simulation results are shown in Figure 4. Where: x_1 continuously increases over time, which is the position response characteristic curve of T; y_1, y_2 continuously shakes vigorously without stopping is the response characteristic curve of H's shaking angle and L's shaking angle. This is a complex double pendulum phenomenon that makes the ability to locate the T position inaccurate and the strong swing angle continuously causes unsafe operation of the GC system. Therefore, the above simulation results are verified to be consistent with the dynamic characteristics of the double pendulum GC.

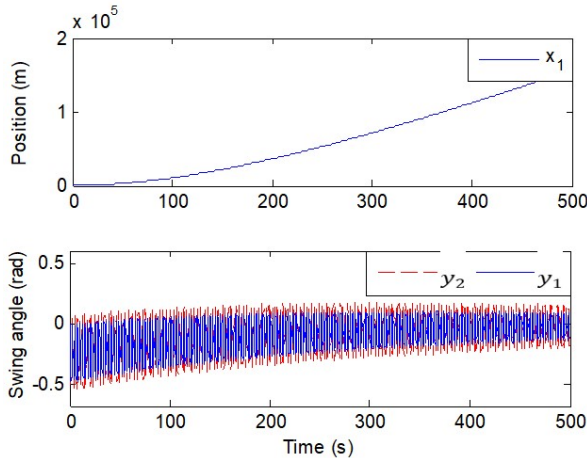


Figure 4. Position response characteristic curve, hook oscillation angle, load oscillation angle

3. DESIGN OF SLIDING CONTROLLER USING LYAPUNOV STABILITY THEORY

The sliding mode controller (SMC) has the advantages of being a nonlinear directional controller that achieves high stability and robustness even when the system is subjected to external disturbances or when the mass, cable length, and position parameters of the GC change over time. The SMC of the GC is designed as follows:

Suppose the actual values of GC position, H's swing angle, L's swing angle are x_1, x_3, x_5 ,

The trolley position, the desired H and L swing angles of the GC system are respectively x_{r1}, x_{r3}, x_{r5} .

The primary control objective of SMC is that under the influence of control signal u , the tracking error between x_1, x_3, x_5 with x_{r1}, x_{r3}, x_{r5} can be converged to 0 when $t \rightarrow \infty$ and the oscillations of the hook and load converge to 0 when $t \rightarrow \infty$. Control deviation is defined as follows:

$$\sigma_n = x_{2n-1} - x_{r(2n-1)} \quad (18)$$

Where: $n = 1, 2, 3$

From the system of state equations of motion of GC (17), the control error (18) the authors have a sliding surface defined for GC with three subsystems as follows:

$$\begin{aligned} s_n &= \dot{\sigma}_n + \tau_n \sigma_n \\ &= x_{2n} + \tau_n (x_{2n-1} - x_{r(2n-1)}) \end{aligned} \quad (19)$$

Where: $n = 1, 2, 3; \tau_n$ are positive real numbers.

To ensure that each subsystem follows its own sliding surface, the control signal u of the entire control rule is defined as follows:

$$u = u_{\sigma q1} + u_{\sigma q} + u_{\sigma q} + u_{sw} \quad (20)$$

Where: u_{sw} is the switching control signal of SMC, $u_{\sigma q1}, u_{\sigma q}, u_{\sigma q3}$ corresponding to the equivalent control laws of the three subsystems, the following expression is obtained:

$$u_{\sigma qn} = -\frac{f_n(X(t)) + \tau_n x_{2n}}{b_n(X(t))} \quad (21)$$

Where: $n = 1, 2, 3$

The general slip surface of SMC is constructed as follows:

$$\begin{aligned} s &= \beta_1 s_1 + \beta_2 s_2 + \beta_3 s_3 \\ &= \beta_1 x_2 + \beta_1 \tau_1 x_{11} + \beta_2 x_4 + \beta_2 \tau_2 x_{33} \\ &\quad + \beta_3 x_6 + \beta_3 \tau_3 x_{55} \end{aligned} \quad (22)$$

Where: $\beta_1, \beta_2, \beta_3$ are positive real numbers.

$$x_{11} = (x_1 - x_{r1}), x_{33} = (x_3 - x_{r3}),$$

$$x_{55} = (x_5 - x_{r5}),$$

Based on Lyapunov stability theory, a positive definite function is selected to design the SMC for controlling GC, as follows:

$$V = \frac{1}{2}s^2 \quad (23)$$

From (23) the derivative of with respect to time yields the following equation:

$$\begin{aligned} \dot{V} &= s\dot{s} = s(\beta_1\dot{s}_1 + \beta_2\dot{s}_2 + \beta_3\dot{s}_3) \\ &= s[\beta_1(\dot{x}_2 + \tau_1\dot{x}_1) + \beta_2(\dot{x}_4 + \tau_2\dot{x}_3) \\ &\quad + \beta_3(\dot{x}_6 + \tau_3\dot{x}_5)] = s[\beta_1(f_1(X(t)) + \tau_1x_2) \\ &\quad + \beta_1b_1(X(t))(u_{\sigma q1} + u_{\sigma q2} + u_{\sigma q3} + u_{sw}) \\ &\quad + \beta_2(f_2(X(t)) + \tau_2x_4) + \beta_2b_2(X(t))(u_{\sigma q1} \\ &\quad + u_{\sigma q2} + u_{\sigma q3} + u_{sw}) + \beta_3(f_3(X(t)) + \tau_3x_6) \\ &\quad + \beta_3b_3(X(t))(u_{\sigma q1} + u_{\sigma q2} + u_{\sigma q3} + u_{sw}) \\ &\quad + \beta_1q_{d1}(X(t)) + \beta_2q_{d2}(X(t)) + \beta_3q_{d3}(X(t))] \\ &= s[\beta_1b_1(X(t))(u_{\sigma q2} + u_{\sigma q3} + u_{sw}) \\ &\quad + \beta_2b_2(X(t))(u_{\sigma q1} + u_{\sigma q3} + u_{sw}) \\ &\quad + \beta_3b_3(X(t))(u_{\sigma q1} + u_{\sigma q2} + u_{sw}) \\ &\quad + \beta_1q_{d1}(X(t)) + \beta_2q_{d2}(X(t)) \\ &\quad + \beta_3q_{d3}(X(t))] \end{aligned} \quad (24)$$

From equation (24) to determine \dot{V} negative, choose \dot{s} for SMC as follows:

$$\dot{s} = -\Delta_s \quad (25)$$

Where: $\Delta_s = k_s \text{sign}(s)$,

$k_s > |\beta_1q_{d1}(X(t)) + \beta_2q_{d2}(X(t)) + \beta_3q_{d3}(X(t))|$ is positive real number,

$$\frac{\Delta_s}{k_s} = \begin{cases} 1 & \text{nếu } s > 0 \\ -1 & \text{nếu } s < 0 \\ 0 & \text{nếu } s = 0 \end{cases} \quad (26)$$

Substituting (25) into (24) yields:

$$\begin{aligned} \dot{V} &= -s\Delta_s + s[\beta_1q_{d1}(X(t)) + \beta_2q_{d2}(X(t)) \\ &\quad + \beta_3q_{d3}(X(t))] \\ &\leq -|s|k_s + |s| * |\beta_1q_{d1}(X(t)) \\ &\quad + \beta_2q_{d2}(X(t)) + \beta_3q_{d3}(X(t))| = -|s|(k_s \\ &\quad - |\beta_1q_{d1}(X(t)) + \beta_2q_{d2}(X(t)) + \beta_3q_{d3}(X(t))|) \\ &< 0 \end{aligned} \quad (27)$$

The results show that the designed SMC can control the GC stably.

From equations (24) and (25), the following is obtained:

$$\begin{aligned} &\beta_1b_1(X(t))(u_{\sigma q2} + u_{\sigma q3} + u_{sw}) \\ &+ \beta_2b_2(X(t))(u_{\sigma q1} + u_{\sigma q3} + u_{sw}) \\ &+ \beta_3b_3(X(t))(u_{\sigma q1} + u_{\sigma q2} + u_{sw}) = \\ &-\Delta_s(28) \end{aligned}$$

From (28), the SMC switching control signal is calculated as follows:

$$\begin{aligned} u_{sw} &= \frac{-1}{\beta_1b_1(X(t)) + \beta_2b_2(X(t)) + \beta_3b_3(X(t))} \\ &* [\beta_1b_1(X(t))(u_{\sigma q2} + u_{\sigma q3}) \\ &+ \beta_2b_2(X(t))(u_{\sigma q1} + u_{\sigma q3}) \\ &+ \beta_3b_3(X(t))(u_{\sigma q1} + u_{\sigma q2}) - \Delta_s] \end{aligned} \quad (29)$$

By substituting (21) and (29) into (24), the sliding mode control rule is designed as follows:

$$\begin{aligned} u &= -(H_1(X))^{-1}(H_2(X) + H_3(X) - \Delta_s) \\ &= -(H_1(X))^{-1}(H_2(X) + H_3(X) + k_s \text{sign}(s)) \end{aligned} \quad (30)$$

Where: $H_1(X) = \beta_1b_1(X(t)) + \beta_2b_2(X(t)) + \beta_3b_3(X(t))$,

$H_2(X) = \beta_1f_1(X(t)) + \beta_2f_2(X(t)) + \beta_3f_3(X(t))$,

$H_3(X) = \beta_1\tau_1x_2 + \beta_2\tau_2x_4 + \beta_3\tau_3x_6$,
 $\tau_1 = 1; \tau_2 = 0,03; \tau_3 = 2,5; \beta_1 = 1;$
 $\beta_2 = 1,69; \beta_3 = 0,026$

From equation (30) shows that the control law of GC contains the function $k_s \text{sign}(s)$. This is the main cause of chattering phenomenon in GC which causes mechanical damage to GC. To eliminate the chattering phenomenon, the authors replaced the function $k_s \text{sign}(s)$ in equation (30) by a value through the fuzzy controller. It follows that:

$$\dot{s} = -\Delta_f \quad (31)$$

Where: $\Delta_f > q_d(X(t))$ is the control law amplitude. To make \dot{V} negative, choose a value of Δ_f that satisfies the condition IF $s > 0$ THEN $\Delta_f > 0$; IF $s < 0$ THEN $\Delta_f < 0$.

$$\Delta_f = \frac{\sum_{j=1}^{\alpha} \omega_{A^j}(s) \times Q^j}{\sum_{j=1}^{\alpha} \omega_{P^j}(s)} \quad (32)$$

Where: P^j and Q^j are fuzzy sets of input variables s and output variables Δ_f , $\omega_A(s)$ is a Gaussian function.

To increase the position tracking efficiency and speed up the convergence, the fuzzy controller k_s is selected to be optimized according to the formula:

$$k_s = |\Delta_{fmax}|/2 \quad (33)$$

Where: $s_{min} = -0.25$; $s_{max} = 0.25$; $\Delta_{fmin} = -10$; $\Delta_{fmax} = 10$; $k_s = 5$

Based on (17) and (30), a double pendulum type GC control scheme using SMC is designed, as shown in Figure 5.

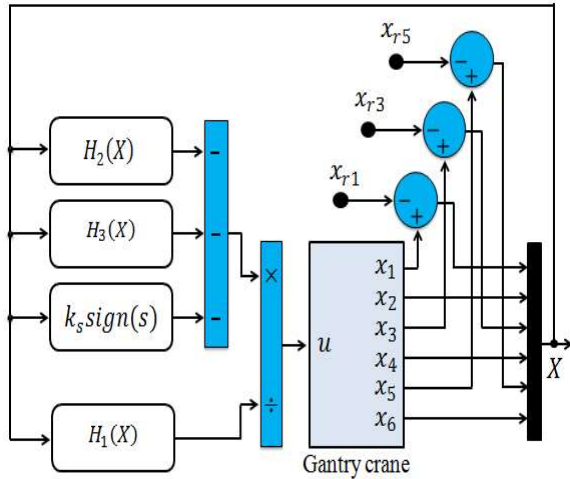


Figure 5. Matlab structure diagram using SMC to control GC

4. SIMULATION RESULTS

Using MATLAB/Simulink software to simulate SMC controlling GC with technical parameters taken from actual values in table 1, $x_{r1} = 10\text{ m}$; $x_{r3} = 0\text{ rad}$; $x_{r5} = 0\text{ rad}$ and $q_d = 0\text{ N}$. The simulation results are shown in Figure 6. Where: The characteristic curve of the position response of T with mass M, the sway angle of H with mass m_1 , The pendulum angle of L has mass m_2 and the corresponding gantry crane control input signal is x_1, y_1, y_2, u , it can be seen that Maximum% overshoot for x_1 (POT) 0%, error $e_{xl} = 0\%$, time to establish position $t_{x1} = 5.2\text{ s}$, H has the largest swing angle $y_{1max} = 0.05\text{ rad}$ and time to establish the swing angle $t_{y1} = 6.5\text{ s}$, As for the swing angle of L, it has the largest angle $y_{2max} = 0.068\text{ rad}$ and time to establish the swing angle $t_{y2} = 6.7\text{ s}$. From the simulation results it can be seen that in the absence of external interference, Double pendulum type GC achieves accurate position in short time 5.2 s , control the oscillation angle of H small 0.05 rad and the oscillation angle of L is small 0.068 rad .

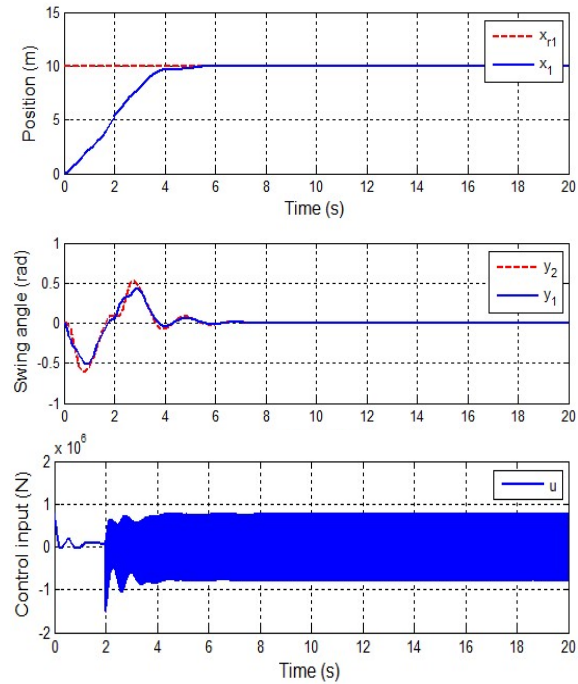


Figure 6. Position response characteristic curve, rocking angle and GC control signal.

In actual production, when operating GC, there are always external disturbances affecting the system. Especially at the time of starting the GC, the increased speed creates great friction, causing the load to oscillate, and combined with the impact of wind and collision, the load shakes more strongly. To test the reliability of SMC, the authors assumed a noise signal [15] in case 1. (C1) is friction $q_{d1} = -1000000\text{ N}$, $t_{d1} = 2\text{ s}$ The impact at GC initiation time with simulation results shown in figure 7 and noise signal in case 2 (C2) is the impact of the load $q_{d2} = 0.5\text{ rad}$, $t_{d2} = 2\text{ s}$ impact on the system at 10s after startup with simulation results shown in figure 8.

The simulation results are shown in Figure 7, Figure 8. Where: $x_{1q}, y_{1q}, y_{2q}, u_q$ respectively, the position response characteristic curve of T, the first sway angle, the second sway angle and the control input signal when there is an impact disturbance still closely follows the characteristic curve x_1, y_1, y_2, u . It can be seen that the system response remains unchanged and good control quality is still achieved.

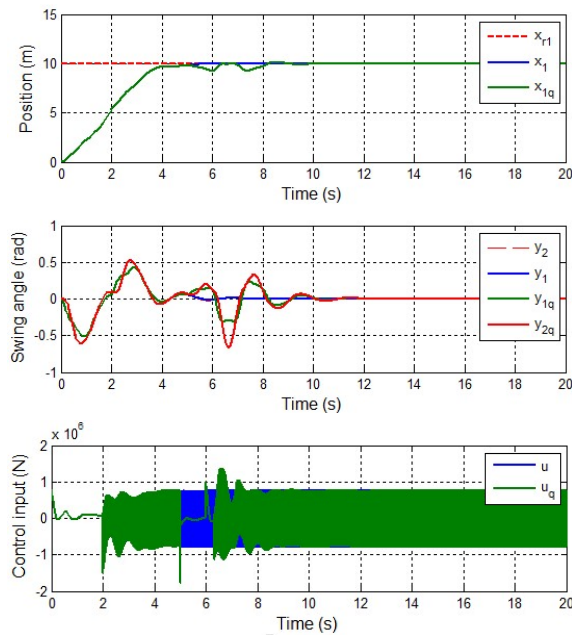


Figure 7. Position response characteristic curve, rocking angle and GC control signal when there is noise
C1

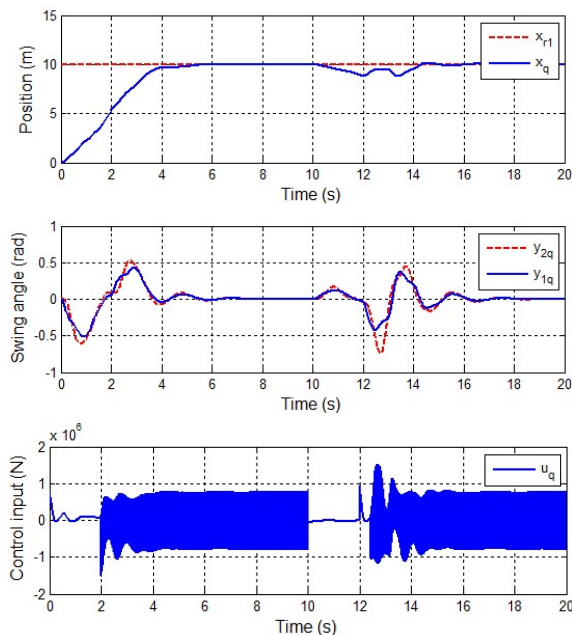


Figure 8. Position response characteristic curve, rocking angle and GC control signal when there is noise
C2

In this study, the GC system under consideration has a significantly larger mass—over 1000 kg greater than those reported in [2], [4], and [14]. Additionally, the travel distance of the forklift in the proposed model is more than 10 times longer than those considered in the aforementioned studies. To clarify the superiority of the solution, the authors

compared the parameters POT (%), e_{xl} (%), t_{x1} (s), t_{y1} (s), t_{y2} (s) of SMC by Lyapunov stability theory and applying FC to select the optimal coefficient k_s with other published control methods as shown in Table 2.

Table 2. Comparison of SMC with other published control methods

| Symbol | SMC | ATC [2] | GA-Fuzzy [4] | Fuzzy [14] |
|------------------|-------------|---------|--------------|------------|
| x_{r1} (m) | 10 | 1 | 1 | 0,8 |
| POT (%) | 0 | 0 | 0 | 0,1 |
| e_{xl} (%) | 0 | 0 | 0 | 0 |
| t_{x1} (s) | 5,2 | 7 | 7,1 | 7,2 |
| t_{y1} (s) | 6,5 | 6,5 | 6,8 | 13 |
| t_{y2} (s) | 6,7 | 6,5 | 6,8 | 13 |
| y_{1max} (rad) | 0,48 | 0,022 | 0,06 | 0,07 |
| y_{2max} (rad) | 0,69 | 0,024 | 0,07 | 0,075 |

Based on the results in Table 2, it can be seen that the controllers all have good control performance. Where: Adaptive tracking control (ATC) [2] has t_{y2} smallest, t_{x1} big. GA-FC [4] and FC [14] all have t_{y1} , t_{y2} , t_{x1} big. Therefore, GC uses the SMC controller that the authors designed to be more optimal than the published control methods.

5. CONCLUSION

In this paper, a dynamic model of GC with double pendulum type is proposed and the accuracy of the dynamic model has been verified through MATLAB/Simulink software. SMC is designed to control GC to move to desired position quickly, while controlling first oscillation angle, second oscillation angle is small. To increase tracking efficiency and accelerate convergence, the control program uses FC to optimize the coefficient k_s of SMC. Based on Lyapunov stability theory, the authors have proven that this system is always stable in the entire GC control space. The efficiency of SMC was tested through MATLAB/Simulink simulation. Simulation results $POT = 0\%$; $e_{xl} = 0$; $t_{x1} = 5,2s$; $y_{1max} =$

$0,05\text{rad}$; $t_{y1} = 6,5\text{s}$; $y_{2\max} = 0,068\text{rad}$; $t_{y2} = 6,7\text{s}$ shows good quality of SMC. To test the reliability of SMC, the authors simulated the impact of interference on GC. The simulation results show that the proposed SMC for GC

control achieves high accuracy, small H-angle and L-angle oscillation. Based on the simulation results, further study of practical applications can be pursued.

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TÓM TẮT

Những khó khăn khi điều khiển hoạt động giàn cần trục (GC) là phải đối diện với hiện tượng phức tạp kiểu con lắc đôi. Sự dao động của móc (H) và tải trọng (L) đã làm giảm hiệu suất làm việc, giảm khả năng định vị chính xác của xe nâng (T), thậm chí gây thiệt hại cơ học và tai nạn mất an toàn. Do đó, bài báo trình bày một giải pháp là thiết kế bộ điều khiển trượt (SMC) để điều khiển giàn cần trục giảm dao động của móc và tải trọng, đồng thời tăng khả năng định vị của xe nâng. Hệ số ks được tối ưu hóa theo bộ điều khiển mờ (FC). Sự ổn định của hệ thống được chứng minh bằng thuyết ổn định Lyapunov, đã đạt được tính chính xác và độ bền của toàn bộ hệ thống điều khiển. Bộ điều khiển trượt đã được kiểm tra thông qua mô phỏng MATLAB/ Simulink. Kết quả mô phỏng cho thấy khi sử dụng bộ điều khiển trượt, hệ thống có chất lượng điều khiển tốt.

Từ khóa: Giàn cần trục, điều khiển trượt, điều khiển vị trí, điều khiển dao động, điều khiển mờ.

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